(B)

1. [3 points] Use logarithmic differentiation to find the derivative of $f(x) = \dfrac{\sin^2 x \arcsin x}{(x^2+4x)^2}$.

$$\ln f(x) = 2\ln \sin x + \ln \arcsin x - 2\ln(x^2+4x)$$

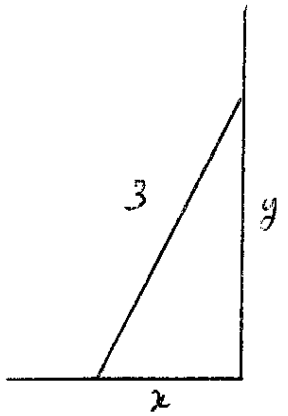
differentiate

$$\frac{1}{f(x)} f'(x) = \frac{2}{\sin x}(\cos x) + \frac{1}{\arcsin x}\left(\frac{1}{\sqrt{1-x^2}}\right) - \frac{2}{x^2+4x}(2x+4)$$

So
$$\boxed{f'(x) = \frac{\sin^2 x \arcsin x}{(x^2+4x)^2}\left(\frac{2\cos x}{\sin x} + \frac{1}{\sqrt{1-x^2}\arcsin x} - \frac{2(2x+4)}{x^2+4x}\right)}$$

(B)

2. [3 points] A ladder 3 m long is leaning against a wall. The top of the ladder is sliding down the wall at 0.5 m/s. How fast is the bottom of the ladder sliding along the floor when the ladder is 2 m from the wall?



$x^2 + y^2 = 9$

$\frac{dy}{dt} = -0.5 \text{ m/s}$

want $\frac{dx}{dt}$ when $x = 2$

if $x = 2$, $(2)^2 + y^2 = 9 \Rightarrow y^2 = 5 \Rightarrow y = \sqrt{5}$

$$\frac{dx}{dt} = -\frac{y}{x}\frac{dy}{dt} = -\frac{\sqrt{5}}{2}(-0.5 \text{ m/s}) = \boxed{\frac{\sqrt{5}}{4} \text{ m/s}}$$

$$\simeq \boxed{0.56 \text{ m/s}}$$

(B)

3. [2 points] Find the absolute maximum and minimum of $f(x) = x - 3\ln x$ on $[1, 4]$.

$f'(x) = 1 - \frac{3}{x}$

$f'(x) = 0$ if $x = 3$

$f(1) = 1$

$f(3) = 3 - 3\ln(3) \simeq -0.2958$

$f(4) = 4 - 3\ln(4) \simeq -0.1589$

$\therefore$ $\boxed{\text{abs max } 1}$ (at $x = 1$)

and $\boxed{\text{abs min } 3 - 3\ln 3}$ (at $x = 3$)

(B)

4. [4 points] Find the following limits:

(a) $\displaystyle\lim_{x\to 0}\frac{e^x-\cos x-x}{x^2}$ (b) $\displaystyle\lim_{x\to 0^+} x^{4x}$

a) $\displaystyle\lim_{x\to 0}\frac{e^x-\cos x-x}{x^2}$ $(0/0)$

$\displaystyle = \lim_{x\to 0}\frac{e^x+\sin x-1}{2x}$ (still $0/0$)

$\displaystyle = \lim_{x\to 0}\frac{e^x+\cos x}{2}$

$= \boxed{1}$

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b) $\displaystyle\lim_{x\to 0^+} x^{4x}$ (0^0)

let $y = x^{4x}$, then $\ln y = 4x\ln x$

$\displaystyle\lim_{x\to 0^+}\ln y = \lim_{x\to 0^+} 4x\ln x = \lim_{x\to 0^+}\frac{4\ln x}{1/x} = 0$

so $\displaystyle\lim_{x\to 0^+} x^{4x} = \lim_{x\to 0^+} y = \lim_{x\to 0^+} e^{\ln y} = e^0 = \boxed{1}$

(B)

5. [2 points] If $f''(x) = 6x + 24x^2 + 3\sin x$, find f.

$$f'(x) = 3x^2 + 8x^3 - 3\cos x + C$$

$$\boxed{f(x) = x^3 + 2x^4 - 3\sin x + Cx + K}$$

(B)

6. [6 points] Sketch the graph of $y = f(x) = \dfrac{x}{x-4}$.

$\lim_{x \to 4^-} \frac{x}{x-4} = -\infty$, $\lim_{x \to 4^+} \frac{x}{x-4} = \infty$ $x = 4$ vertical asymptote

$(0,0)$ is only intercept

$\lim_{x \to -\infty} \frac{x}{x-4} = 1$, $\lim_{x \to \infty} \frac{x}{x-4} = 1$ so $y = 1$ horizontal asymptote

$f'(x) = \frac{-4}{(x-4)^2}$ $f(x)$ always decreasing, no extrema

$f''(x) = \frac{8}{(x-4)^3}$, $x < 4$ $f''(x) < 0$ $f(x)$ concave down

$x > 4$ $f''(x) > 0$ $f(x)$ concave up

but no inflection points